We express our gratitude to our students for their diligent recollection of these questions, listed here in no specific order. Although some phrasings may deviate from the original, their essence remains intact.

1) Given that a_n is the remainder of $7^n \div 11$, find $a_1 + a_2 + a_3 + \cdots + a_{2024}$

Notice that

$$7 \equiv 7 \ (mod\ 11)$$

$$7^2 \equiv 5 \ (mod\ 11)$$

$$7^3 \equiv 7^2 \times 7 \equiv 5 \times 7 \equiv 35 \equiv 2 \ (mod\ 11)$$

$$7^4 \equiv 7^3 \times 7 \equiv 2 \times 7 \equiv 14 \equiv 3 \ (mod\ 11)$$

$$7^5 \equiv 7^4 \times 7 \equiv 3 \times 7 \equiv 21 \equiv 10 \ (mod\ 11)$$

$$7^6 \equiv 7^5 \times 7 \equiv 10 \times 7 \equiv 70 \equiv 4 \ (mod\ 11)$$

Hang on! These patterns usually terminate at some point….

$$7^7 \equiv 7^6 \times 7 \equiv 4 \times 7 \equiv 28 \equiv 6 \ (mod\ 11)$$

$$7^8 \equiv 7^7 \times 7 \equiv 6 \times 7 \equiv 42 \equiv 9 \ (mod\ 11)$$

$$7^9 \equiv 7^8 \times 7 \equiv 9 \times 7 \equiv 63 \equiv 8 \ (mod\ 11)$$

$$7^{10} \equiv 7^9 \times 7 \equiv 8 \times 7 \equiv 56 \equiv 1 \ (mod\ 11)$$

And there is no need to go any further as the next residue will be 7, thus repeating the cycle.

Hence, the remainders follow a pattern of 7, 5, 2, 3, 10, 4, 6, 9, 8, 1 with 10 terms.

$$\frac{2024}{10} = 202\ r\ 4$$

So the answer is

$$202 * (7 + 5 + 2 + 3 + 10 + 4 + 6 + 9 + 8 + 1) + (7 + 5 + 2 + 3)$$

$$202 * 55 + 17 = 11,127$$

Answer: 11,127

2) Find the sum of $x + y$ in the sequence $9, 24, 69, 204, 609, x, y$

Observe that the values increase dramatically, so there is some multiplication involved. By inspection, it can be observed that each following term is the previous term $\times 3 - 3$.

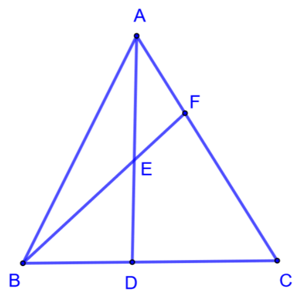
$$x = 609 \times 3 - 3 = 1827 - 3 = 1824$$

$$y = 1824 \times 3 - 3 = 5472 - 3 = 5469$$

$$x + y = 1824 + 5469 = 7293$$

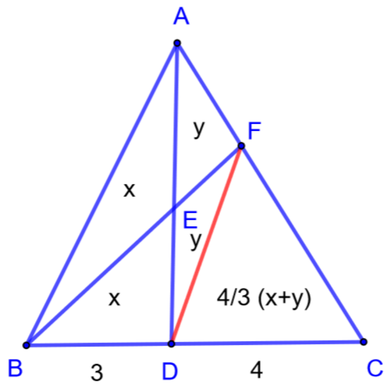
Answer: 7293

3) Given that $BD{:}DC = 3{:}4$ and E is the mid-point of AD. Given that $\Delta AEF + \Delta BED = 30$, find the area of ΔABC

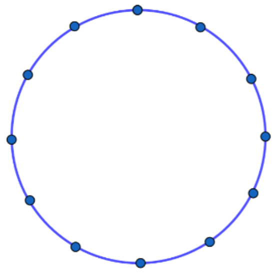


1) First draw the line DF.
2) As E is the midpoint of AD, let $y = \Delta AFE = \Delta DEF, x = \Delta ABE = \Delta BDE$, so $x + y = 30$
3) $\Delta BDF{:}\Delta CDF = 3{:}4$, hence
$$\Delta CDF = \frac{4}{3}(x+y)$$
4) And so the area of ΔABC is
$$2x + 2y + \frac{4}{3}(x+y)$$
$$= 60 + \frac{4}{3}(30) = 100$$

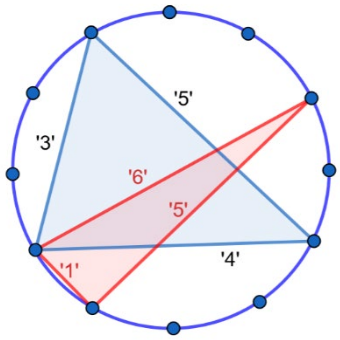
Answer: 100



4) Given the 12 points a circle as shown in the diagram, how many distinct triangles can be created using any 3 points as vertices? Flipped or rotated triangles are considered the same.



Triangles in this circle as be encoded using the length of their spans. For example, the blue triangle is a 3-4-5 while the red triangle is 1,6,5.



There cannot be that many triangles, so we can simply bash it.

1+1+10	1+5+5	2+5+5
1+2+9	2+2+8	3+3+6
1+3+8	2+3+7	3+4+5
1+4+7	2+4+6	4+4+4

Answer: 12

5) What is the sum of digits of the numerator and denominator of

$$\frac{1}{2^2-1}+\frac{1}{4^2-1}+\frac{1}{6^2-1}+\cdots+\frac{1}{2024^2-1}$$

This is a classic telescoping series question. While it is not difficult to see that the first few terms are

$$\frac{1}{3}+\frac{1}{15}+\frac{1}{35}\cdots$$

Using the difference of squares formula…we can find out the last term

$$a^2-b^2=(a+b)(a-b)$$

$$2024^2-1=2024^2-1^2=(2024+1)(2024-1)=2025\times 2023$$

$$\frac{1}{3}+\frac{1}{15}+\frac{1}{35}+\cdots+\frac{1}{(2023)(2025)}$$

$$=\frac{1}{1\times 3}+\frac{1}{3\times 5}+\frac{1}{5\times 7}+\cdots+\frac{1}{2023\times 2025}$$

$$=\frac{1}{2}\left(\frac{1}{1}-\frac{1}{3}+\frac{1}{3}-\frac{1}{5}+\frac{1}{5}-\frac{1}{7}+\cdots+\frac{1}{2023}-\frac{1}{2025}\right)$$

$$=\frac{1}{2}\left(\frac{1}{1}-\frac{1}{2025}\right)=\frac{1}{2}\left(\frac{2024}{2025}\right)=\frac{1012}{2025}$$

$$1+1+2+2+2+5=13$$

Answer: 13

6) Calculate

$$\left(\frac{1}{2}+\frac{1}{3}+\cdots+\frac{1}{31}\right)+\left(\frac{2}{3}+\frac{2}{4}+\cdots+\frac{2}{31}\right)+\cdots\left(\frac{29}{30}+\frac{29}{31}\right)+\frac{30}{31}$$

Grouping by denominator,

$$\frac{1}{2}+\left(\frac{1}{3}+\frac{2}{3}\right)+\left(\frac{1}{4}+\frac{2}{4}+\frac{3}{4}\right)+\left(\frac{1}{5}+\frac{2}{5}+\frac{3}{5}+\frac{4}{5}\right)+\cdots+\left(\frac{1}{31}+\frac{2}{31}+\cdots\frac{30}{31}\right)$$

$$\frac{1}{2}+\frac{2}{2}+\frac{3}{2}+\frac{4}{2}+\cdots+\left(\frac{30\times 31}{2}\times\frac{1}{31}\right)$$

$$\frac{1}{2}(AP(1\ to\ 30)=\frac{1}{2}\left(\frac{30\times 31}{2}\right)=\frac{930}{4}=\frac{465}{2}=232.5$$

Answer: 232.5

7) Sam was calculating $(x+y)\div z$. He typed $x+y\div z$ on the calculator and got 15. He then typed $y+x\div z$ and got 20. If the calculator evaluates $\div$ before $+$, what will the actual answer for $(x+y)\div z$?

In equation form, the calculator's behaviour can be expressed as

$$\frac{y}{z}+x=15 \qquad (1)$$

$$\frac{x}{z}+y=20 \qquad (2)$$

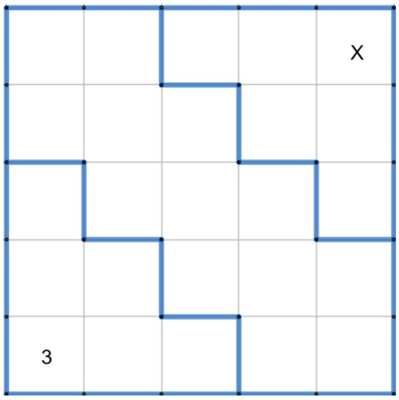
But wait! Do we really want to try to use some complicated technique for a question that you really have only 3 minutes for? I would say, let's just guess, if only because the numbers 15 and 20 are both rather small.

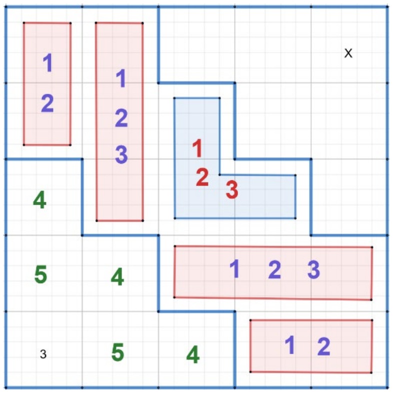
Notice that both x and y can be divided by z, so let's make a guess that they are both divisible by both 2 and 3, and hence 6. Thus, we can guess that x = 12, and thus $\frac{y}{z}$ has to be 3. Also, from the equations, you can tell that y is probably bigger than x.

I got it on my 3rd try that x=12, y=18 and z=6 and so $\frac{x+y}{z}=5$

Answer: 5

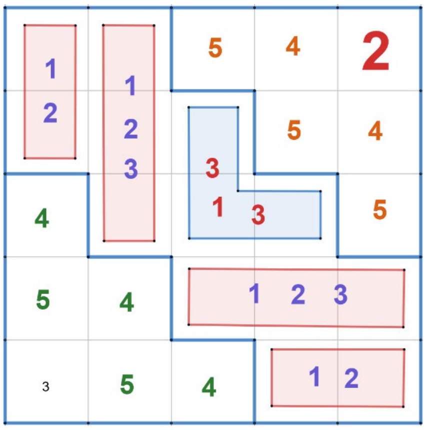
8) Given that each number 1,2,3,4,5 appears only once in each row and column and that the sum of the numbers in each of the three segmented areas is the same, find x.





First notice that since each row has a sum of $1+2+3+4+5=15$, the total sum of all numbers in the entire diagram is $15*5=75$, and hence each zone has a total of $75 \div 3 = 25$.

Bottom Left Zone: First looking at the bottom zone, we can see that we need to fill in a total of $25-3=22$ in 5 boxes. $22 \div 5 > 4$ and thus, we can reasonably deduce that the 5 empty boxes must be all 4s or 5s. Some testing will lead to the conclusion that the arrangement of numbers as shown in the diagram is the only possibility.



Middle Zone: We can also determine that the top 2 boxes in column 1 and the top 3 boxes in column 2 must be some permutation of 1+2 and 1+2+3 respectively. Likewise for the rightmost 3 boxes on row 4 and rightmost 2 boxes on row 5.

At this point, we realise that we are still short of $25 - (1+2+1+2+3) \times 2 = 7$ to be filled across 3 boxes. Using Sudoku rules, you can easily tell that there are no 4s or 5s involved. The only way to do this is to use 3-3-1.

Top Right Zone: At this point, realise that all "1"s, "2"s and "3"s have already been used. The top right zone must contain 3 "5"s (since only 2 were used at the bottom left zone), and the only way to arrange 3 "5"s in the top right zone is shown in the diagram. (You can do it easily if you are familiar with Sudoku)

We know we are short of another 2 "4s" and a single "2" and the answer "2" is apparent.

Answer: 2

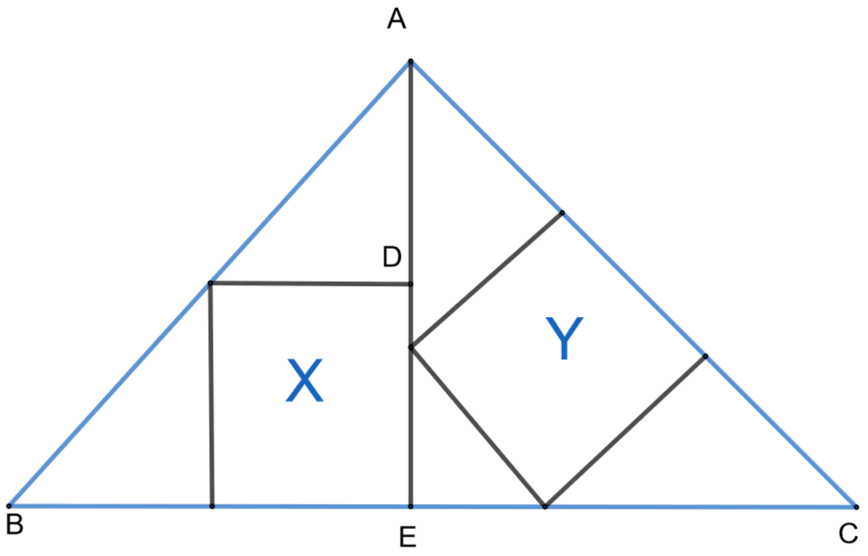
9) A and B are travelling from X to Y, and Y to X respectively. If A increases speed by 15% and B increases by 12km/h, then they will meet at the same point where they would have if their speeds were not changed. What is B's speed?

This is very straightforward. 15% of B's speed is equal to 12km/h, so B's speed is

$$\frac{12}{15} \times 100 = 80$$

Ans: 80km/h

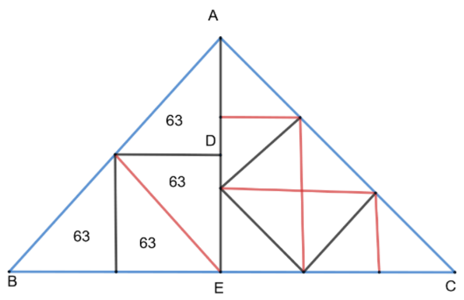
10) Triangle ABC is isosceles and the area of square X = 126. Given that A intersects the midpoint of BC at E, ADE is a straight line, find the area of square Y.



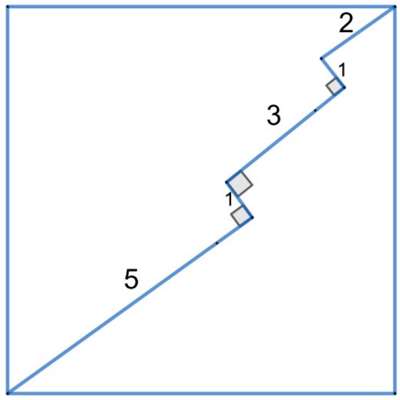
This is a traditional slice-it-up question. 126/2 = 63, so each slice on the left is 63, giving a total of $63 \times 4 = 252$ for half the triangle.

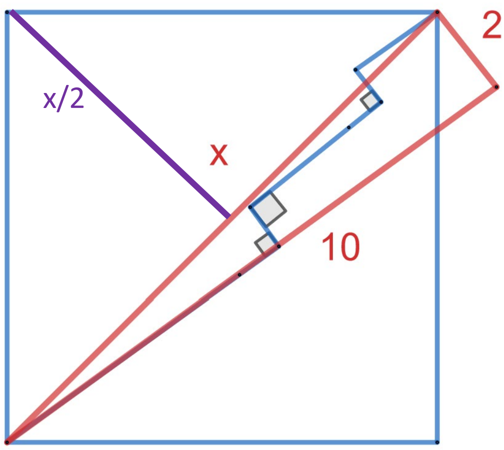
And since the right side of triangle ABC can be sliced up into 9 segments, each segment is $252 \div 9 = 28$ and thus the area of square Y is $28 \times 4 = 112$.

Answer: 112



11) Find the area of the square.





It is like the 10th time I have seen this question or its variants (Common on Facebook Geometry groups). It's a direct Pythagoras question once you can see the right-angled triangle hiding inside.

$$x^2 = 10^2 + 2^2 = 104$$

$$Area\ of\ square = 2 \times \frac{1}{2} \times x \times \frac{x}{2}$$

$$Area\ of\ square = \frac{x^2}{2} = \frac{104}{2} = 52$$

Answer: 52

12) There are 3 identical bottles of orange juice. Bottle A is $\frac{2}{3}$ concentrate + $\frac{1}{3}$ water, bottle B is $\frac{4}{7}$ concentrate $\frac{3}{7}$ water, and bottle C is $\frac{7}{12}$concentrate $\frac{5}{12}$ water. If they are all mixed together, what is the ratio of juice to water?

We could have done this as a basic ratio question (the usual ratio method), but since the 3 bottles are identical, we merely need to add up the concentrate fractions.

$$\frac{2}{3} + \frac{4}{7} + \frac{7}{12} = \frac{56 + 48 + 49}{84} = \frac{153}{84} = \frac{51}{28}$$

This concentrate amount is based on 3 bottles, so we need to divide by 3, giving us

$$\frac{51}{28} \times \frac{1}{3} = \frac{17}{28}$$

$\frac{17}{28}$ is concentrate and of course $1 - \frac{17}{28} = \frac{11}{28}$ is water.

Answer: 17:11

13) If $[1,2,3,\dots,n]$ represent the least common multiples of 1,2,3,..., n, then how many distinct results are there in the following 99 terms?

$$\frac{[1,2]}{2!}, \frac{[1,2,3]}{3!}, \frac{[1,2,3,4]}{4!}, \dots, \frac{[1,2,\dots,100]}{100!}$$

Working out the first few terms, we see that

$$\frac{[1,2]}{2!} = 1, \quad \frac{[1,2,3]}{3!} = 1, \quad \frac{[1,2,3,4]}{4!} = \frac{1}{2}, \quad \frac{[1,2,3,4,5]}{5!} = \frac{1}{2}, \quad \frac{[1,2,3,4,5,6]}{6!} = \frac{1}{12}$$

Also observe that whenever a prime number is introduced in the numerator, the answer does not change (it is cancelled out by its factorial counterpart)

There are 24 prime numbers between 3 and 99 inclusive, and hence $99 - 24 = 75$.
Answer: 75

14) Alice, Bob and Charlie have some marbles. If Alice gives 20 marbles to Charlie, the ratio of (Alice + Bob) to Charlie's marbles is 1:2. If Alice gives Bob 30 marbles, the ratio of (Alice + Charlie) to Bob's marbles is 1:3. What is the total number of marbles they have?

It feels that we can either use the usual ratio methods or algebra to solve this.

$$2[A - 20 + B] = 2A - 40 + 2B = C + 20$$

$$3[A - 30 + C] = 3A - 90 + 3C = B + 30$$

$$2A + 2B = C + 60 \qquad (1)$$

$$3A + 3C = B + 120 \qquad (2)$$

$(1) \times 3$

$$6A + 6B = 3C + 180 \qquad (3)$$

$(2) + (3)$

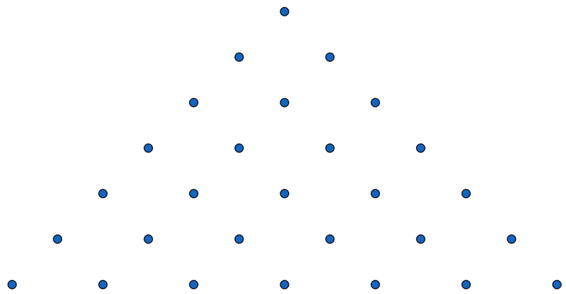
$$9A + 5B = 300$$

Notice that Alice gave Bob 30 marbles, so A is at least 30. Substituting A=30 into the equations, we quickly get $A = 30, B = 6, C = 12$

$$30 + 6 + 12 = 48$$

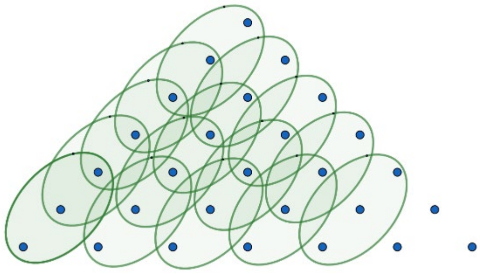
Answer: 48

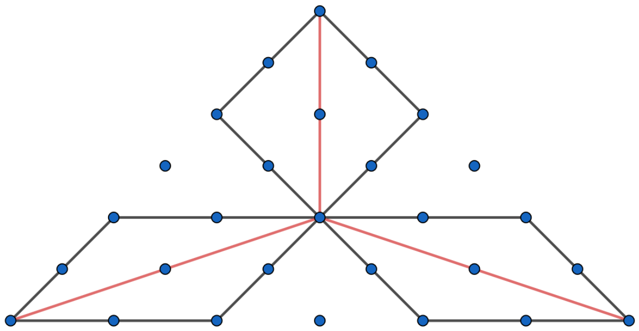
15) How many line segments pass through exactly 3 points?



Given 7 points on each side, we can see that there will be $(1+2+3+4+5) = 15$ such segments for each side of the triangle.

$$(1+2+3+4+5) \times 3 = 45$$





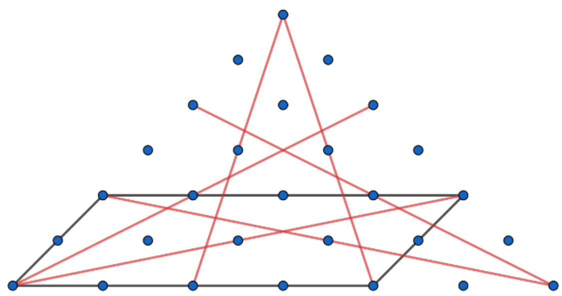
The tricky part about this question is that there are some other ways of drawing a 3-point segment, specifically, they are hidden inside the 2x2 and 2x4 parallelograms. (This is when you start singing the parallelogram song so loudly that everybody wonders what happened to you!)

There are 3 types of 2x2 parallelograms that you can form, and each carries a brand new 3-point segment. Counting, you can tell that there are 6 instance of each parallelogram types

$$6 \times 3 = 18$$

Next, we go for the 2x4 parallograms and we can find 6 of them, each with a new 3-point segment.

$$45 + 18 + 6 = 69$$



Answer: 69

16) There are 19 cards in a pile numbered 1 to 19. You draw the cards randomly. What is the minimum number of cards you must draw to ensure you definitely have 2 cards that sum to 20?

Notice that Card No 10 is useless. It cannot be paired with any other cards to get 20.

The remaining cards can be split into 2 groups, the cards 1 to 9 and the cards 11 to 19.

Group A cards: 1, 2 ,3 ,4 ,5 ,6, 7, 8, 9

Group B cards: 11, 12, 13, 14, 15, 16, 17, 18, 19

You cannot obtain 20 if you have cards from only 1 group, but if you have ALL the cards from either group, any single card from the other group will give you a pair of cards that add to 20. And don't forget Mr Card 10 who is destined to be lonely in this question.

$$9 + 1 + 1 = 11$$

Answer: 11

17) If the sum of N consecutive whole numbers is 2024, what is the largest possible value of N?

Using the AP formula

$$\frac{n}{2}(2a + (n-1)d)$$

$d = 1$ since the numbers are consecutive

$$2024 = \frac{n}{2}(2a + n - 1)$$

Multiplying by 2 to get rid of the troublesome fraction, we get

$$4048 = n(2a + n - 1)$$

And we know that $4048 = 2^4 \cdot 11 \cdot 23$

When n is even, $(2a + n - 1)$ will become odd as $(2\times even + even - 1)$ is odd.

So $n = 2^4 = 16$, and $(2a + n - 1) = 11 \times 23 = 253$

When n is odd, $(2a + n - 1)$ will be even as $(2 \times odd + odd - 1)$is even.

So $n = 23$, and the other part becomes 176.

(We cannot use $n = 23 \times 11 = 253$ as that will cause the sequence to involve negative numbers)

Answer: 23

18) There are some members in a band. In 2023, there were 30 more boys than girls. In 2024, the number of band members increased by 10%. The number of girls increased by 20% while the number of boys increased by 5%. How many band members are there now (2024)?

This type of question is readily solved by using a table. Notice that there is a "5%" in the question and if we use a bigger initial value for the number of girls, we can avoid some troublesome looking decimals.

	Boys	Girls	Total
2023	100G+30	100G	200G+30
2024	105G+31.5	120G	220G+33

Using the 2024 row, we can form the equation:

$$105G + 31.5 + 120G = 220G + 33$$

Solving, we get

$$225G = 220G + 33 - 31.5$$

$$5G = 1.5$$

$$G = 0.3$$

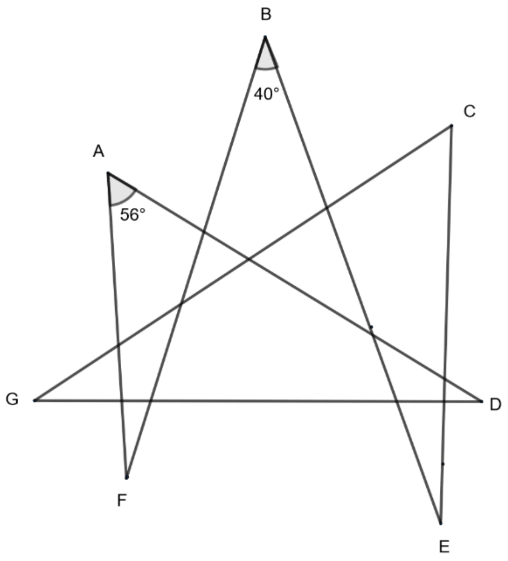
Substituting G=0.3 into 220G+33, we get

$$220(0.3) + 33 = 66 + 33 = 99$$

Answer: 99

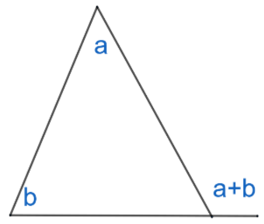
19) It appears that everybody has a question that is different from others??

20) Find $\angle C + \angle D + \angle E + \angle F + \angle G$



There are undoubtedly many ways to solve this type of questions.

Our solution for this question makes extensive use of the fact that the exterior angle of a triangle is the sum of the 2 remote interior angles. It's really just a shortcut.



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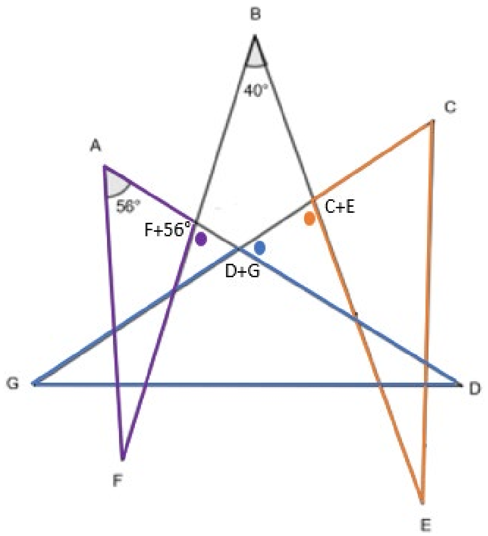
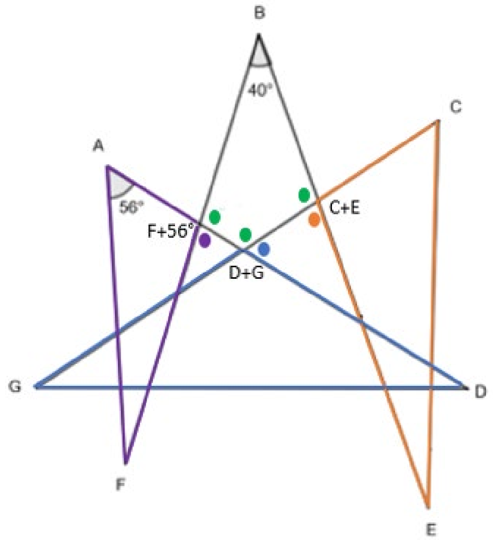


Step 2:



From the last diagram, it can be seen that all the desired angles are now grouped into three straight lines.

The three green angles = 360° – 40° = 320°

$$\angle C + \angle D + \angle E + \angle F + \angle G + 56^\circ = 3 \times 180^\circ - 320^\circ$$

$$\angle C + \angle D + \angle E + \angle F + \angle G = 220^\circ - 56^\circ$$

$$\angle C + \angle D + \angle E + \angle F + \angle G = 164^\circ$$

Answer: 164°

Last updated: 27 March