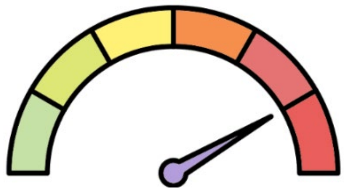
avocado lab
math & science

NMOS 2024

FULL SOLUTIONS

you can do it!



Question	Mark	Correct Answer	My Answer	Score
1	1	228		
2	1	60		
3	1	17		
4	1	3135		
5	1	660		
6	1	102		
7	1	96		
8	1	52		
9	1	240		
10	1	58		
11	2	725		
12	2	19		
13	2	7		
14	2	8		
15	2	900		
16	2	30		
17	2	5		
18	2	100		
19	2	3		
20	2	98		
21	3	30		
22	3	368		
23	3	288		
24	3	4		
25	3	14		
26	3	1400		
27	3	46		
28	3	10		
29	3	208		
30	3	20		
31	4	441		
32	4	440		
33	4	1230		
34	4	1960		
35	4	13		

Questions 1 to 10 are worth 1 mark each

1. Evaluate

$$(19 \times 19 - 12 \times 12) \div (\frac{19}{12} - \frac{12}{19})$$

Answer

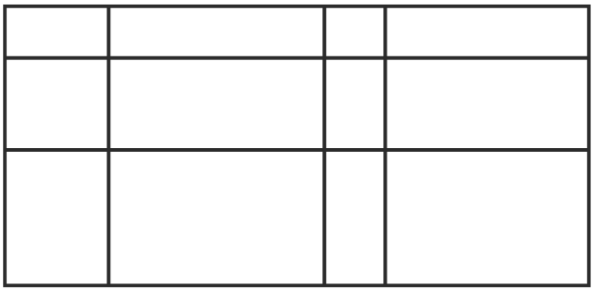
228

Solution: When solving this type of equation problems, there is no need to rush in and compute the exact values. Instead, try to see if there is some general form that can be simplified.

$$(a^2 - b^2) \div \left(\frac{a}{b} - \frac{b}{a}\right) = (a^2 - b^2) \div \left(\frac{a^2 - b^2}{ab}\right) = ab$$

$$ab = 19 \times 12 = 228$$

2. Find the number of rectangles (including squares) in the following diagram.



Answer

60

Solution: You could have solved this by brute force but it would be time consuming and error prone. You can solve it more elegantly by noticing that each rectangle has top, bottom, left and right wall.

$$\binom{4}{2}\binom{5}{2} = 6 \cdot 10 = 60$$

3. There are altogether 30 coins of 20 cents and 50 cents with a total worth of $9.90. How many 20-cent coins are there?

Answer

17

Solution: This type of questions can often be solved easily by re-writing it in equation form.

Let k be the number of 20-cent coins, then

$$20k + 50(30 - k) = 990$$

$$20k + 1500 - 50k = 990$$

$$30k = 510$$

$$k = 17$$

4. Given that a, b, c and d are different prime numbers, and $a + b + c = d$, find the smallest possible value of $a \times b \times c \times d$.

Answer

3135

Solution: Listing out the first few prime numbers will quicky yield

$$3 + 5 + 11 = 19$$

$$3 \times 5 \times 11 \times 19 = 3135$$

5. Town A and Town B are connected by a straight road. At 9am, car X travels from Town A to Town B at a constant speed, and Car Y travels from Town B to Town A at a constant speed. They meet at the place 30 meters away from the midpoint of the two towns on this road. Given that the speed of car X is $\frac{5}{6}$ of the speed of car Y, find the distance between Town A and Town B in meters.

Answer

660

Solution: If the ratio of the speeds is 5 : 6, then the distance travelled by the 2 cars is 5 : 6, and the midpoint is at the 5.5u mark. This means that 0.5u=30m.

$$11 \times 2 \times 30 = 660$$

6. $\overline{ABC}$ and $\overline{DEF}$ represent two 3-digit numbers where A, B, C, D, E and F are distinct digits, and A and D are non-zero. In the following sum, what is the smallest $\overline{ABC}$?

$$\begin{array}{r} A\ \ B\ \ C \\ +\,D\ \ E\ \ F \\ \hline 5\ \ 6\ \ 7 \end{array}$$

Answer

102

Solution: WHAT?! For this question, you could have simply tried from 102 (the smallest possible ABC with distinct digits) and found that it works……

Otherwise, just do the usual guess and check.

7. A tin of cookies was shared among Alan, Barbara and Connie in the ratio of 4 : 6 : 14. If the cookies were shared equally among them instead, Barbara would have 8 more pieces of cookies. Determine the total number of cookies in the tin.

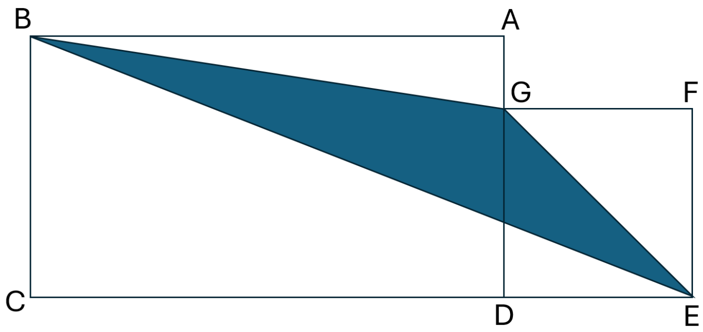
Answer

96

Solution: Free mark question. 4+6+14=24, there are 24 units altogether. 24/3=8, so Barbara would have 8 units if the cookies are shared equally. An increase of 2 units for Barbara = 8 cookies.

$$24 \times 4 = 96$$

8. The diagram below shows a rectangle $ABCD$ and a square $DEFG$ side by side. The area of the rectangle is $150\ cm^2$ with the ratio of length to width being 3 : 2. The area of the square is $64\ cm^2$. Find the area of the shared region in cm^2.



Answer

52

Solution: Some quick calculations will tell you that the length and width of the rectangle are 15 and 10 respectively. So just take the area of the whole diagram and subtract the white space.

$$150 + 64 - 32 - \frac{15 \times 2}{2} - \frac{10 \times 23}{2} = 182 - 15 - 115 = 52$$

9. Mark and James can assemble 80 toy trains together in two hours. Mark alone can assemble 32 toy trains in 80 minutes. Assuming their rate of working remain constant throughout, how many minutes does James alone take to assemble a total of 64 toy trains?

Answer

240

Solution: Mark can assemble 32/8 = 4 trains in 10 minutes, so Mark can assemble $4 \times 12 = 48$ trains in 2 hours. This means that James can assemble $80 - 48 = 32$ trains in 2 hours and of course he can assemble 64 trains in twice the time.

10. In a class of 40 students, the average score for a Mathematics test is 76. Two of the students, Alicia and Betty leave the class. The average score of the remaining students increases to 77. Given that Alicia scores 2 more marks than Betty, what is Alicia's score?

Answer

58

Solution: You can do this the traditional way or by logic.

Traditional way:

Total score for the class = $40 \times 76 = 3040$

New total score for the class = $38 \times 77 = 2926$

Decrease = $3040 - 2926 = 114$

Alicia's score $= \frac{114-2}{2} + 2 = 58$

Logic:

Their departure brings the average score of the entire score up by 1 mark

Their total score was 76+76-38=114

Alicia's score = (same as above)

Questions 11 to 20 are worth 2 mark each

11. Sammy placed an online order for 275 plates and some cups for her new café. When her order was delivered, she inspected and found that 8% of her plates and 4% of the cups were chipped. The other items, made up of 94.9% of the total number of items ordered, were delivered in good condition. How many cups did Sammy buy?

Answer

725

Solution: This is an equation type question that needs to be tackled step by step systematically. Quite irritating, but it's still nothing compared to NMOS 2013 Preliminary Round Question 21, the really evil vegetable and meat question.

$$Number\ of\ chipped\ plates = 275 \times 0.08 = 22$$

Let C be the total number of cups

$$Number\ of\ chipped\ cups = 4\% \times C = \frac{C}{25}$$

Total % of items that are chipped = 100-94.9 = 5.1%

$$\frac{5.1}{100}(275 + C) = 22 + \frac{C}{25}$$

$$5.1\,(275 + C) = 2200 + 4C$$

$$1402.5 + 5.1C = 2200 + 4C$$

$$1.1C = 797.5$$

$$C = 725$$

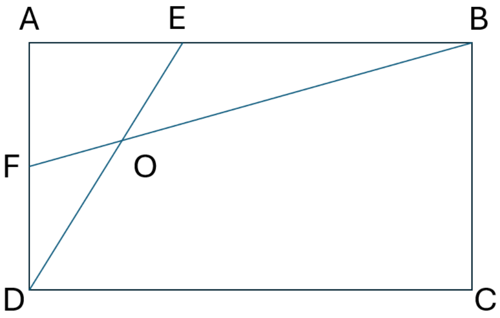
12. A palindrome is a number that reads the same backwards and forwards, such as 1001 and 54345. Find the smallest 2-digit number AB such that the sum $AB + BA$ is not a palindrome.

Answer

19

Solution: Notice that AB+BA = 11 (A+B), and all 2 digit solutions to this are palindromes. Hence we are looking for the smallest A+B that gives a 3 digit solution to 11(A+B), and of course we get A+B=10.

13. In the diagram below, area of rectangle $ABCD$ is 60 cm^2. Given that $BE = 2AE$ and $AF = FD$, find the area of quadrilateral $AEOF$ in cm^2.



Answer

7

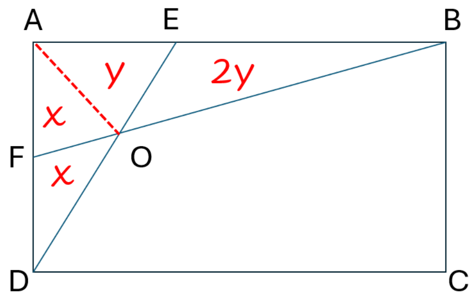
Solution: This is a popular question that has come out in similar forms in many Asian MOs. First draw the dotted line AO.

Using the ratios provided in the question, we can determine that

$$\Delta ABF = \frac{1}{4} \times 60 = 15 = x + 3y$$

$$\Delta ADE = \frac{1}{6} \times 60 = 10 = 2x + y$$

Solving for x & y,



$$x = 3, y = 4$$
$$x + y = 7$$

14. At a running race, competitors A, B and C are running at constant speeds. A is 100 meters ahead of B and B in turn is 300 meters ahead of C. B is confident he can catch up with A in 5 mins while C is certain that he can catch up with B in 10 mins. How many minutes does C take to catch up with A?

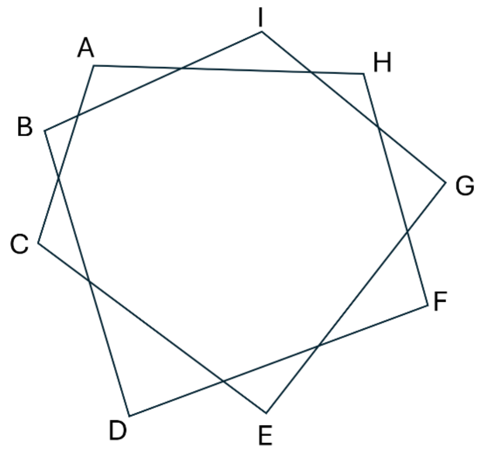
Answer

8

Solution: Let's look at the differences in speeds among the competitors

$$B - A = \frac{100}{5} = 20\ m/min$$
$$C - B = \frac{300}{10} = 30m/min$$
$$C - A = 20 + 30 = 50m/min$$
$$\frac{400}{50} = 8\ mins$$

15.In the diagram below, find $\angle A + \angle B + \angle C + \angle D + \angle E + \angle F + \angle G + \angle H + \angle I$ in degrees.



Answer
900

Easy Solution: This question is a perfect candidate for the 'Spin your pencil" method!

Solution: This is really tough to explain on paper in a short manner, but here's the gist of it.

Sum of all angles in the interior 9-gon

$$(9 - 2)180 = 7 \times 180$$

Notice that all our desired angles are inside one of the 9 triangles. Let's call the other 2 angles in each triangle "stuff".

$$Stuff = 2(180 \times 9 - 180 \times 7) = 4 \times 180$$

$$What\ we\ want = 9 \times 180 - Stuff = 900$$

Honestly, just use the Spin your Pencil method.......

16. It is known that the sum and the difference of two whole numbers are in the ratio 29 : 9. Furthermore, the product of these two whole numbers is 1710. Find the smaller number among these two numbers.

Answer

30

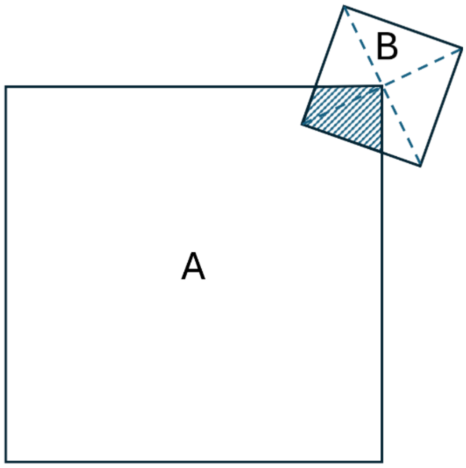
Solution: Prime factorise 1710 straight away and you will get

$$1710 = 2^1 3^2 5^1 19^1$$

Given the ratio of the sum and difference, you can easily deduce that the two numbers are in a 19:10 ratio, approximately 2:1. Looking at the prime factors, it should be easy to obtain

$$A = 19 \times 3, \quad B = 2 \times 3 \times 5 = 30$$

17. In the following diagram, squares A and B are of different sizes. The intersection point of diagonals in B coincides with a vertex point of A. It is known that the overlapping area (shown as the shaded region is $\frac{1}{9}$ of the area of square A. If the length of a side of square A and the length of a side of square B are in the ratio of $m : n$, where $\frac{m}{n}$ is in its simplest form, find the value of $m + n$.



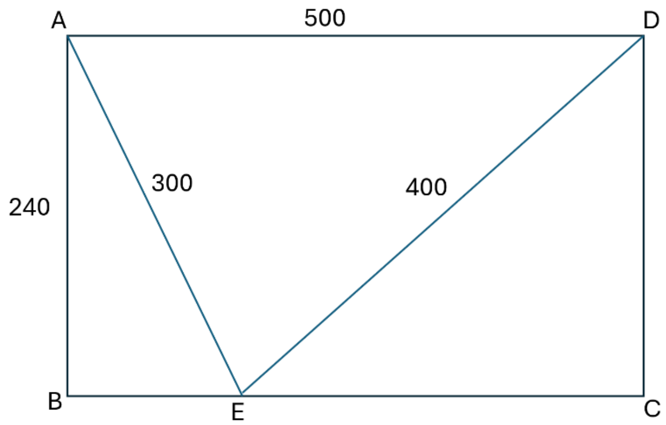
Answer

5

Solution: Since $\frac{1}{4}$ of square B is shared, let's assume that area to be 1 and hence the area of square B is 4 → side of B = 2

And since $\frac{1}{9}$ of square A is shaded, then the area of square A is 9 and the side of square A is 3.

$$3 + 2 = 5$$



18. In the diagram below, $ABCD$ is a rectangle with length of 500 meters and width of 240 meters. E is a point on BC such that AE = 300 meters and DE = 400 meters. Peter travels along the rectangle via the route A→B→C→D→A, and then rest for 20 seconds at point A. Quintus also travels along the triangle via the route A→E→D→A without resting. If both travel at the speed of 1 meter per second and start travelling from A at the same time, find the time taken (in mins) that they will next meet again at A.

Answer

100

Solution: Peter takes 240+500+240+500+20=150 seconds to travel one round. (Don't forget the 20 seconds!)

Quintus takes 300+400+500=1200 seconds to travel one round.

$$LCM(1500,1200) = 6000secs = 100\ minutes$$

19. In the calendar of a certain month, there are 5 Mondays, 5 Tuesdays and 5 Wednesdays. On which day is the last day of the month?

If your answer is Monday, then share your answer as "1";

If your answer is Tuesday, then share your answer as "2";

If your answer is Wednesday, then share your answer as "3";

If your answer is Thursday, then share your answer as "4";

If your answer is Friday, then share your answer as "5";

If your answer is Saturday, then share your answer as "6";

If your answer is Sunday, then share your answer as "7";

Answer

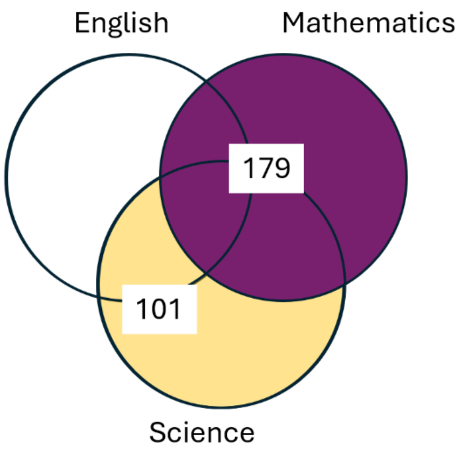
3

Solution: Just draw a calendar and tick 5 Mondays, 5 Tuesdays and 5 Wednesdays and you will see that 31 days have already been allocated and hence Wednesday must be the last day of the month.

20. In a certain school, 172 students passed English, 179 students passed Mathematics, and 202 students passed Science but failed Mathematics. Given that all 378 students in the school passed at least one of the three abovementioned subjects, how many students passed English only?

Answer

98



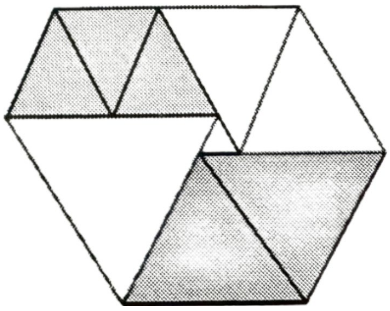
Solution: This is a classic Venn diagram problem.

Plugging in the numbers, you can see that

$$378 - 179 - 101 = 98$$

Questions 21 to 30 are worth 3 mark each

21. The 6-sided figure below is composed of 9 equilateral triangles. Given that the side of the smallest equilateral triangle in the middle is 1cm, find the perimeter of the 6 sided figure in cm.

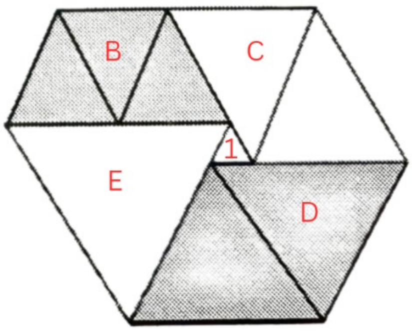


Answer

30

Solution: There will be a temptation to simply draw lines and slice the diagram up. However, since no other information is available, that strategy may not yield the correct answer.

To be very safe, let's name the different types of triangles, 1, B, C, D and E.



Notice that

$$2B = E$$
$$C = B + 1$$
$$D = C + 1$$
$$E = D + 1$$

Thus,

$$E = B + 3 = 2B$$
$$B = 3$$
$$C = 4, D = 5, E = 6$$

$$3 + 3 + 4 + 4 + 5 + 5 + 6 = 30$$

22. There are 11 numbers in a certain sequence, as shown below:

$$a, b, c, d, e, f, g, h, i, j, k$$

The sum of these 11 numbers is 2024. If the difference between any two consecutive terms is a constant, i.e.,

$$b - a = c - b = d - c = e - d = f - e = g - f = h - g = i - h$$
$$= j - i = k - j$$

What is the value of $a + k$?

Answer
368

Solution: Just use the AP formula in reverse

$$\frac{(a + k) \times 11}{2} = 2024$$
$$(a + k) \times 11 = 4048$$
$$a + k = \frac{4048}{11} = 368$$

23. From a solid cube with each side being 6cm long, Figure 2, which is composed of 7 identical cubes, is removed so that each face has a 2cm by 2cm square hole. The remaining figure is Figure 1. Find the total surface are of Figure 1 in cm^2.

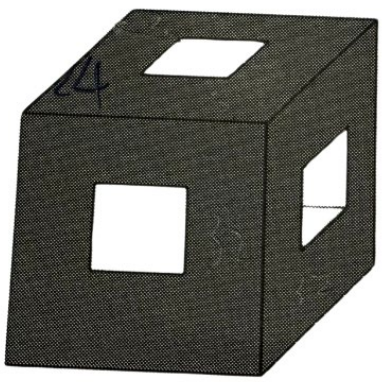


Figure 1

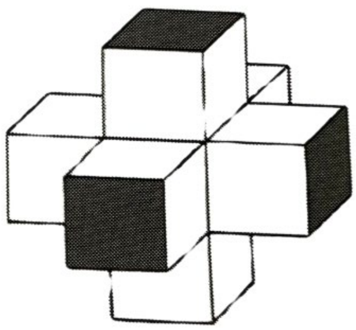


Figure 2

Answer
288

Solution: This diagram is similar to RIPMWC 2009 Round 1 Question 1.

The total surface area of Figure 1 is all the shaded area in Figure 1 + all the unshaded area of Figure 2.

$$Shaded\ in\ Figure\ 1 = (6^2 - 2^2) \times 6 = 32 \times 6 = 192$$
$$Unshaded\ in\ Figure\ 2 = 24 \times 4 = 96$$
$$192 + 96 = 288$$

24. Let us generate a sequence of numbers by multiplying 2 to the preceding term to get the next term, with 2 as the first term, as follows:

$$2, 4, 8, 16, 32, 64, 128, , 256, \ldots$$

What is the remainder when the sum of the 40th term and the 67th term of this sequence is divided by 5?

Answer

4

Solution: Notice that every 4th power of 2 mod 5 is 1. E.g. $2^4 = 16, 16\ mod\ 5\ \equiv 1\ or\ 2^8 = 256\ and\ 256\ mod\ 5 \equiv 1$

Hence the 40th term $mod\ 5\ \equiv 1$, and so is the 64th term (or you can work backwards from the 68th term but that can be more confusing so we prefer to do it forward)

Thus, the 65th term is $mod\ 5 \equiv 2$, the 66th is $mod\ 5\ \equiv 4$, and the 67th is $mod\ 5 \equiv 8 \equiv 3$.

$$1 + 3 = 4$$

25. In a certain population,

- The ratio of the number of women who wear glasses to the number of women who do not wear glasses is 5 : 8,
- The ratio of the number of men who wear glasses to the number of men who do not wear glasses is 3 : 4,
- The ratio of the number of people who wear glasses to the number of people who do not wear glasses is 7 : 10.

If the ration of the number of women who wear glasses to the number of men who wear glasses is $m:n$, where $\frac{m}{n}$ is in its simplest form, find the value of $m+n$.

Answer

14

Solution: Drawing a simple table for such questions can help a lot.

	Glasses	No Glasses
Women	5x	8x
Men	3y	4y

Using the 7 : 10 ratio, we form the following equation.

$$10(5x+3y) = 7(8x+4y)$$
$$50x+30y = 56x+28y$$
$$6x = 2y$$
$$3x = y$$

Substitute 3x=y back into the table and we get $5x$ women wearing glasses and $3 \times 3 = 9x$ men wearing glasses

$$5+9=14$$

26. A furniture retailer had a total of 7270 dining table and sofa sets for sale at the start of the month. He sold $\frac{3}{5}$ of his dining tables and 72% of his sofa sets. He imported an additional 52 dining tables. In the end, the number of sofa sets he had left was $16\frac{1}{3}$% of the number of dining tables left. How many sofa sets did he have at first?

Answer

1400

Solution: It's useful to work backwards for this question.

In the end, the ratio of sofa to dining tables left is

$$16\frac{1}{3}\,x : 100\,x$$
$$49\,x : 300\,x$$

This means that the initial number of sofa sets and dining tables is

$$\frac{49x}{28} \times 100 : \frac{300x - 52}{2} \times 5$$
$$175\,x : 750x - 130$$

The total number of furniture is 7270 (given in question), so

$$175x + 750x - 130 = 7270$$
$$925x = 7400$$
$$x = 8$$

The initial number of sofa sets is 175x, so

$$175 \times 8 = 1400$$

27. A particular brand of detergent is sold in three different sizes at a supermarket as shown in the table below:

	Amount of detergent per packet	Cost per packet
Small	2 litres	\$2.50
Medium	5 litres	\$6
Large	9 litres	\$10

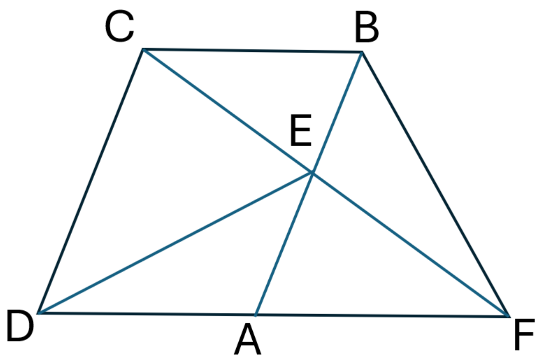
What is the maximum number of litres of the detergent that can be purchased with \$52?

Answer
46

Solution: Notice that the 9 litres pack is the best value of money and so we will want to maximise the number of it. A bit of trial and error should yield

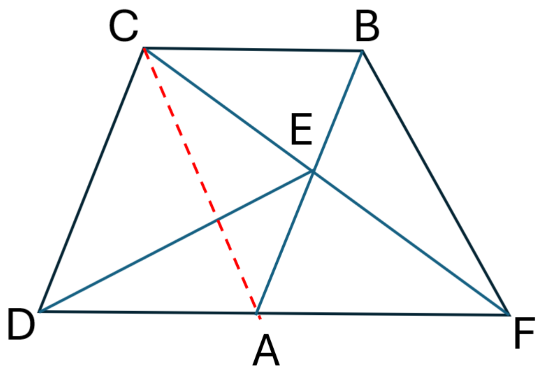
$$4 \times 9 + 5 \times 2 = 46$$

28. In the diagram below, DAF is a straight line and $ABCD$ is a parallelogram with CF and AB intersecting at E. If the area of ΔADE is $10\ cm^2$, find the area of ΔBEF in cm^2



Answer

Solution: Draw the line CA.



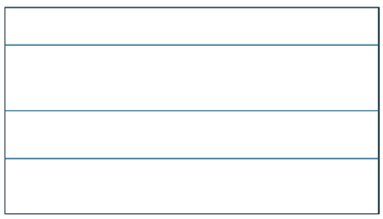
$$\Delta ACE = 10$$
$$\Delta ABC = 10 + \Delta CBE = \Delta FBC$$
$$\Delta FBC = \Delta CBE + \Delta BEF$$
$$\Delta BEF = 10$$

29. There are 20 straight lines drawn on a plane, among which 3 lines are parallel to each other. What is the maximum number of regions that can be divided by these 20 lines?

Answer
208

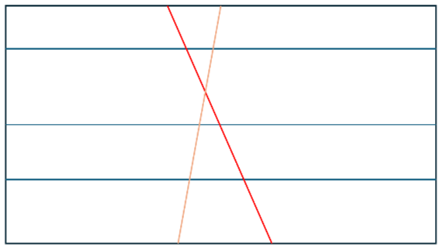
Solution: This question can be solved by simulation. You can imagine that the plane is a rectangle.

First draw the 3 parallel lines and you can see that 4 regions are created.



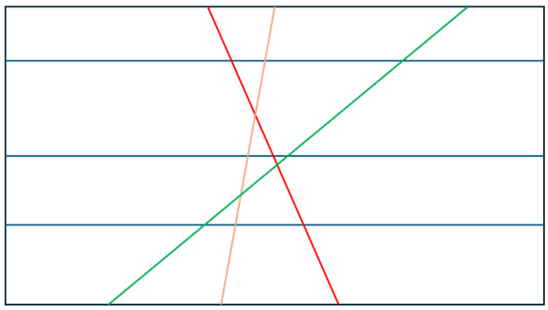
Next realise that the 4th line drawn introduces 4 new regions.

The 5th line introduces 5 new regions.



When drawing lines for such questions, remember that every new line drawn must cross every single existing line (and not hit any existing intersections) in order for the number of regions to be maximised.

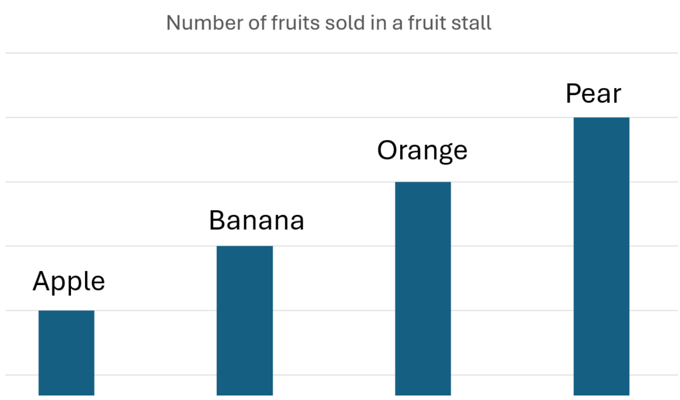
Drawing the 6th line, you can tell that it introduces 6 new regions.



Hence, the number of regions created by 20 lines is

$$4 + 4 + 5 + 6 + 7 \ldots + 20 = 208$$

30. A fruit seller sells only apples, bananas, oranges and pears. The bar chart below shows the number of fruits sold in his fruit stall in the month of January. However the bottom part of the bar chart is torn off. If pears made up 30% of his January sales of fruits and x% of the fruits sold are apples, find the value of x.



Answer

20

Solution: We can represent the number of fruits sold as an equation, where a is the number of apples sold.

$$a + (a + d) + (a + 2d) + (a + 3d) = 4a + 6d$$

Since pears make up 30%,

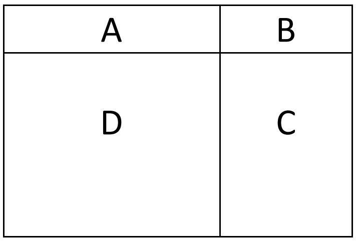
$$10 \times (a + 3d) = 3\,(4a + 6d)$$
$$10a + 30d = 12a + 18d$$
$$12d = 2a$$
$$a = 6$$

The number of fruits are 6,7,8,9.

$$\frac{6}{6 + 7 + 8 + 9} = \frac{6}{30} = 20\%$$

<u>Questions 31 to 35 are worth 3 mark each</u>

31. In the rectangle below, figure A, B and C are rectangles and figure D is a square. If the sum of areas of A and B is $23cm^2$ and the sum of areas of B and C is $44\ cm^2$, given that all length of rectangles are positive integers, find the maximum area of the square D in cm^2.



Answer

441

Solution: Given that $23 = 1 \times 23, 44 = 1 \times 44\ or\ 2 \times 22$, we can quickly determine that $A + B = 1 \times 23$ and that $B + C = 2 \times 22$. The side of square D is this 23-2 or 22-1 = 21

$$21^2 = 441$$

32. Let us consider the product of two numbers, where X and y are positive whole numbers:

$$(1+\frac{2}{x})(1+\frac{2}{y})$$

In some cases, the product above produces a number of the form $1+\frac{1}{z}$ for some positive whole numbers z, such as the following:

$$\left(1+\frac{2}{8}\right)\left(1+\frac{2}{30}\right)=1+\frac{1}{3}$$

$$\left(1+\frac{2}{18}\right)\left(1+\frac{2}{40}\right)=1+\frac{1}{6}$$

Find the smallest product xy such that

$$(1+\frac{2}{x})\left(1+\frac{2}{y}\right)=1+\frac{1}{5}$$

Answer

440

Solution: Simplifying the equation

$$\left(\frac{2+x}{x}\right)\left(\frac{2+y}{y}\right)=\frac{6}{5}$$

$$5(x+2)(y+2)=6xy$$

$$10x+10y+20=xy$$

$$xy-10x-10y-20=0$$

$$(x-10)(y-10)=120$$

$$x=20, y=22$$

$$18\times 22=440$$

33. Malik purchases a total of 12 fruits, consisting of both apples and oranges, for \$14.70. One apple costs \$0.03 more than one orange and Malik bought more apples than oranges. How much, in cents, did Malik pay for the apples alone?

Note: The smallest denomination is 1 cent. There are no fraction of cents.

Answer

1230

Solution: Given that each apple is \$0.03 more than an orange, it tells that the number of apples must be some multiple of 5, otherwise the total sum cannot be a multiple of 10 cents.

And since there are more apples than oranges, the only possibility is that there are 10 apples and 2 oranges.

Now, we can use a simple equation to solve the question.

$$12x + 10(3) = 1470$$
$$12x = 1440$$
$$x = 120$$
$$120 + 3 = 123$$

Each apple costs 123 cents.

$$123 \times 10 = 1230$$

34. A number is formed by writing "1" after every n "0"s, with the number of "0"s written increased by one more after writing each "1" (where n= 0,1,2,3,...). The first 36 digits of this number is show below.

110100100010000100000100000010000000

Determine the number of "0"s in the first 2024 digits

Answer

1960

Solution: We can see this as groups of

$$1 \quad 10 \quad 100 \quad 1000 \; 10000 \; ....$$

We need to make a small guess for a AP that is close to 2024. It turns out that

$$\frac{63 \times 64}{2} = 2016$$

This means that there are 2016 digits up to the 63th group. Among the next 8 digits, there is a single "1" from the 64th group. Thus, the number of "0" is

$$2024 - 64 = 1960$$

35. A meteorological report shows the following records of a certain number of days .

- It rained 12- times, either in the morning, afternoon or evening (For clarity, if rained the entire day, this is counted as having rained 3 times in total).
- There were 12 dry mornings.
- There were 13 dry afternoons.

What is the minimum possible number of days the meteorologist could have recorded?

Answer

13

Solution: The restriction here comes from the rainy slots. If there are 13 dry afternoons, then the minimum number of days is at least 13.

And if there are 13 days, there are at least 13 evenings where it can rain. And hence 13 is the answer.